7.5 Applications from Science and Statistics

Work Revisited

EXAMPLE 1 Finding the Work Done by a Force

Find the work done by the force $F(x) = \cos(\pi x)$ newtons along the x-axis from $x = 0$ meters to $x = 1/2$ meter.

EXAMPLE 2 Work Done Lifting

A leaky bucket weighs 22 newtons (N) empty. It is lifted from the ground at a constant rate to a point 20 m above the ground by a rope weighing 0.4 N/m. The bucket starts with 70 N (approximately 7.1 liters) of water, but it leaks at a constant rate and just finishes draining as the bucket reaches the top. Find the amount of work done

(a) lifting the bucket alone;

(b) lifting the water alone;

(c) lifting the rope alone;

(d) lifting the bucket, water, and rope together.

EXAMPLE 3 Work Done Pumping

The conical tank in Figure 7.42 is filled to within 2 ft of the top with olive oil weighing 57 lb/ft^3. How much work does it take to pump the oil to the rim of the tank?

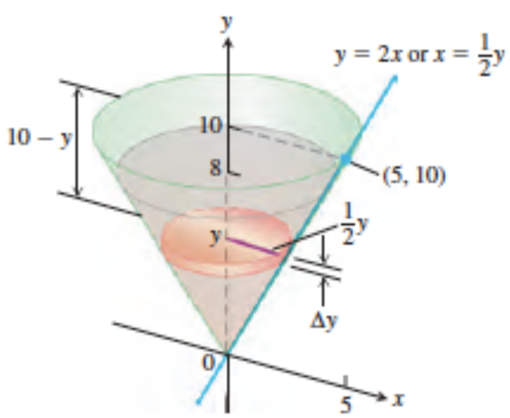


Figure 7.42 The conical tank in Example 3.

Fluid Force and Fluid Pressure

In any liquid, the **fluid pressure** p (force per unit area) at depth h is

$$p = wh,$$

Dimensions check: $\frac{\text{lb}}{\text{ft}^2} = \frac{\text{lb}}{\text{ft}^3} \times \text{ft}$, for example

where w is the *weight-density* (weight per unit volume) of the liquid.

EXAMPLE 4 **The Great Molasses Flood of 1919**

$p = wh$

At 1:00 P.M. on January 15, 1919 (an unseasonably warm day), a 90-ft-high, 90-foot-diameter cylindrical metal tank in which the Puritan Distilling Company stored molasses at the corner of Foster and Commercial streets in Boston's North End exploded. Molasses flooded the streets 30 feet deep, trapping pedestrians and horses, knocking down buildings, and oozing into homes. It was eventually tracked all over town and even made its way into the suburbs via trolley cars and people's shoes. It took weeks to clean up.

(a) Given that the tank was full of molasses weighing 100 lb/ft^3, what was the total force exerted by the molasses on the bottom of the tank at the time it ruptured?

(b) What was the total force against the bottom foot-wide band of the tank wall (Figure 7.44)?

a) $p = 100\,\frac{lb}{ft^3} \cdot 90\,f = 9000\,\frac{lb}{ft^2}$ (force/area)

(force) force/area · area

$F(x) = 9000\,\frac{lb}{ft^2}\left(\pi (45)^2\right)$

$\approx 57{,}225{,}526\ lb$

b) $F(x) = \underset{p}{100\,y_k} \cdot \underset{a}{(90\pi\,\Delta y)} = 9000\pi\, y_k\,\Delta y$ (circumference · h)

$= \int_{89}^{90} 9000\pi y\, dy$

$9000\pi \int_{89}^{90} y\, dy$

$9000\pi \left(\frac{y^2}{2}\right)\Big|_{89}^{90}$

$= 2{,}530{,}553\ lb$

Normal Probabilities

Suppose you find an old clock in the attic. What is the probability that it has stopped somewhere between 2:00 and 5:00?

DEFINITION Probability Density Function (pdf)

A **probability density function** is a function $f(x)$ with domain all reals such that

$$f(x) \geq 0 \text{ for all } x \quad \text{and} \quad \int_{-\infty}^{\infty} f(x)\,dx = 1.$$

Then the probability associated with an interval $[a, b]$ is

$$\int_a^b f(x)\,dx.$$

Probabilities of events, such as the clock stopping between 2:00 and 5:00, are integrals of an appropriate pdf.

EXAMPLE 5 Probability of the Clock Stopping

Find the probability that the clock stopped between 2:00 and 5:00.

$f(t) = \frac{1}{12}$ $\quad 0 \leq t \leq 12$

0 $\quad$ otherwise

$$\int_2^5 f(t)\,dt$$

$$\downarrow$$

$$\frac{1}{12}\,dt$$

$$= \frac{1}{12}t \Big|_2^5$$

$$= \frac{1}{4}$$

DEFINITION Normal Probability Density Function (pdf)

The **normal probability density function (Gaussian curve)** for a population with mean μ and standard deviation σ is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}.$$

The 68-95-99.7 Rule for Normal Distributions

Given a normal curve,

- 68% of the area will lie within σ of the mean μ,
- 95% of the area will lie within 2σ of the mean μ,
- 99.7% of the area will lie within 3σ of the mean μ.

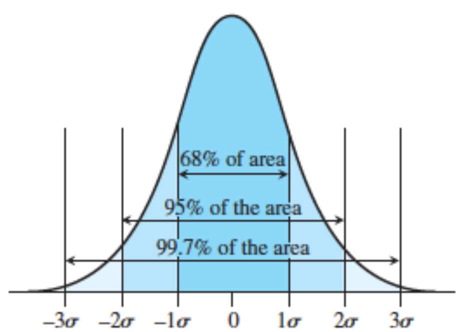
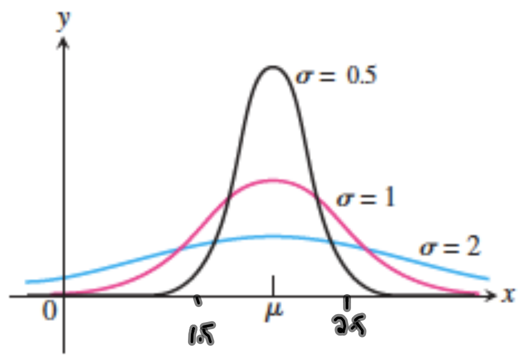


Figure 7.48 The 68-95-99.7 rule for normal distributions.

Even with the 68-95-99.7 rule, the area under the curve can spread quite a bit, depending on the size of σ. Figure 7.49 shows three normal pdfs with mean $\mu = 2$ and standard deviations equal to 0.5, 1, and 2.



EXAMPLE 6 A Telephone Help Line

Suppose a telephone help line takes a mean of 2 minutes to answer calls. If the standard deviation is $\sigma = 0.5$, then 68% of the calls are answered in the range of 1.5 to 2.5 minutes and 99.7% of the calls are answered in the range of 0.5 to 3.5 minutes.

EXAMPLE 7 Weights of Spinach Boxes

Suppose that frozen spinach boxes marked as "10 ounces" of spinach have a mean weight of 10.3 ounces and a standard deviation of 0.2 ounce.

(a) What percentage of *all* such spinach boxes can be expected to weigh between 10 and 11 ounces?

(b) What percentage would we expect to weigh less than 10 ounces?

(c) What is the probability that a box weighs *exactly* 10 ounces?

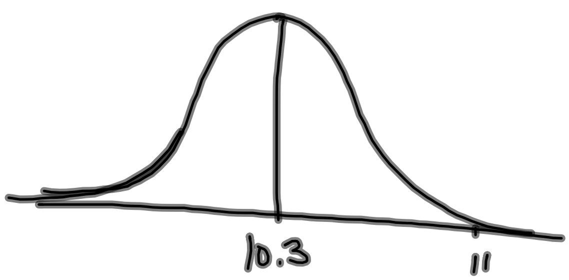
$\mu = 10.3$
$\sigma = .2$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

a) $$\int_{10}^{11} \frac{1}{.2\sqrt{2\pi}} e^{-(x-10.3)^2/(2\cdot .2^2)}$$

$\approx .93$

b)



$$\int_{9}^{10} \frac{1}{.2\sqrt{2\pi}} e^{-(x-10.3)^2/(2\cdot .2^2)}$$

$\approx .067$ about 7%

c) $\int_{10}^{10} f(x)\,dx = 0$